Hong Kong Mathematics Olympiad (1997 – 98)

Final Event (Group) – Sample Question

香港数学竞赛 (1997 – 98)

决赛项目 (团体) – 样本题目

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特别声明，答案须用数字表达，并化至最简。

(i) 若 a 是最小的正整数被 3 除时余 1 而能被 5 整除，求 a。

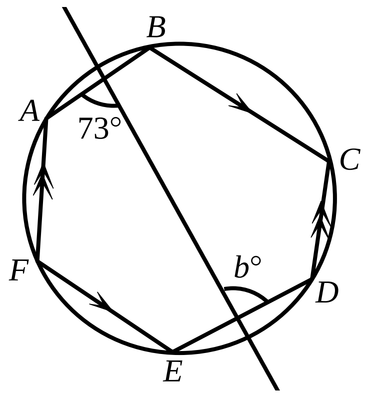
If a is the smallest positive integer which gives remainder 1 when divided by 3 and is a multiple of 5, find a.

$a =$

(ii) 下图中， $FA \mathbin{/\!/} DC$ 及 $FE \mathbin{/\!/} BC$，求 b。

In the following diagram, $FA \mathbin{/\!/} DC$ and $FE \mathbin{/\!/} BC$. Find b.

$b =$



(iii) 若 c 是一两位正整数，其两位之和是 10 而两位之积是 25。求 c。

If c is a 2 digit positive integer such that sum of its digits is 10 and product of its digits is 25. Find c.

$c =$

(iv) 若 S_1, S_2, …, S_{10} 是一由正整数组成的 A.P. 的头十项使得

$S_1 + S_2 + \cdots + S_{10} = 55$ 及 $(S_{10} - S_8) + (S_9 - S_7) + \cdots + (S_3 - S_1) = d$。求 d。

If S_1, S_2, …, S_{10} are the first ten terms of an A.P. consisting of positive integers such that $S_1 + S_2 + \cdots + S_{10} = 55$ and

$(S_{10} - S_8) + (S_9 - S_7) + \cdots + (S_3 - S_1) = d$. Find d.

$d =$

Hong Kong Mathematics Olympiad (1997 – 98)

Final Event 1 (Group)

香港数学竞赛 (1997 – 98)

决赛项目 1 (团体)

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特别声明，答案须用数字表达，并化至最简。

(i) 若扇形面积 $s = 4\text{ cm}^2$，扇形半径 $r = 2\text{ cm}$ 及扇形的弧长 $A = p\text{ cm}$，求 p 的数值。

$p =$

If the area of a given sector $s = 4\text{ cm}^2$, the radius of this sector $r = 2\text{ cm}$ and the arc length of this sector $A = p\text{ cm}$, find the value of p.

(ii) 已知 $\dfrac{a}{2b+c} = \dfrac{b}{2c+a} = \dfrac{c}{2a+b}$ 且 $a+b+c \neq 0$。若 $q = \dfrac{2b+c}{a}$，求 q 的数值。

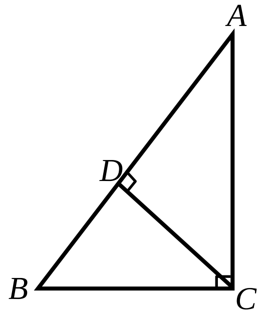
$q =$

Given that $\dfrac{a}{2b+c} = \dfrac{b}{2c+a} = \dfrac{c}{2a+b}$ and $a+b+c \neq 0$. If $q = \dfrac{2b+c}{a}$, find the value of q.

(iii) 设直角三角形 ABC 中，CD 是斜边 AB 上的高，$AC = 3$，$DB = \dfrac{5}{2}$，$AD = r$，求 r 的数值。

$r =$

Let ABC be a right-angled triangle, CD is the altitude on AB, $AC = 3$, $DB = \dfrac{5}{2}$, $AD = r$, find the value of r.



(iv) 若 $x^3 + px^2 + qx + 17 \equiv (x+2)^3 + a$，求 a 的数值。

$a =$

If $x^3 + px^2 + qx + 17 \equiv (x+2)^3 + a$, find the value of a.

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Final Event 2 (Group)

香港数学竞赛 **(1997 – 98)**

决赛项目 **2 (**团体**)**

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除非特别声明，答案须用数字表达，并化至最简。

(i) 若 $\frac{137}{a}=0.1\dot{2}3\dot{4}$，求 a 的数值。

If $\frac{137}{a}=0.1\dot{2}3\dot{4}$, find the value of a.

$a =$

(ii) 若 $b=1999\times19981998-1998\times19991999+1$，求 b 的数值。

If $b=1999\times19981998-1998\times19991999+1$, find the value of b.

$b =$

(iii) 若参数方程 $\begin{cases} x=\sqrt{3-t^2} \\ y=t-3 \end{cases}$ 可转换为 $x^2+y^2+cx+dy+6=0$，求 c 及 d 的数值。

If the parametric equations $\begin{cases} x=\sqrt{3-t^2} \\ y=t-3 \end{cases}$ can be transformed into $x^2+y^2+cx+dy+6=0$, find the values of c and d.

$c =$

$d =$

Hong Kong Mathematics Olympiad (1997 – 98)

Final Event 3 (Group)

香港数学竞赛 (1997 – 98)

决赛项目 3 (团体)

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特别声明，答案须用数字表达，并化至最简。

(i) 在 ΔABC 中，$\angle ABC = 2\angle ACB$，$BC = 2AB$。若 $\angle BAC = a°$，求 a 的数值。

In ΔABC, $\angle ABC = 2\angle ACB$, $BC = 2AB$. If $\angle BAC = a°$, find the value of a.

$a =$

(ii) 已知 $x + \frac{1}{x} = \sqrt{2}$，$\frac{x^2}{x^4 + x^2 + 1} = b$，求 b 的数值。

Given that $x + \frac{1}{x} = \sqrt{2}$, $\frac{x^2}{x^4 + x^2 + 1} = b$, find the value of b.

$b =$

(iii) 若方程 $x + y + 2xy = 141$ 有 c 个正整数解，求 c 的数值。

If the number of positive integral root(s) of the equation $x + y + 2xy = 141$ is c, find the value of c.

$c =$

(iv) 已知 $x + y + z = 0$、$x^2 + y^2 + z^2 = 1$ 及 $d = 2\left(x^4 + y^4 + z^4\right)$，求 d 的数值。

Given that $x + y + z = 0$, $x^2 + y^2 + z^2 = 1$ and $d = 2\left(x^4 + y^4 + z^4\right)$, find the value of d.

$d =$

Hong Kong Mathematics Olympiad (1997 – 98)

Final Event 4 (Group)

香港数学竞赛 **(1997 – 98)**

决赛项目 **4 (**团体**)**

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特别声明，答案须用数字表达，并化至最简。

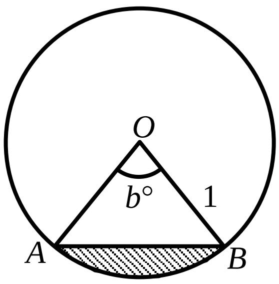
(i) 若 $0.\dot{1}+0.0\dot{2}+0.00\dot{3}+\cdots+0.00000000\dot{9}=a$，求 a 的数值。
(答案以小数表示)。

If $0.\dot{1}+0.0\dot{2}+0.00\dot{3}+\cdots+0.00000000\dot{9}=a$, find the value of a.
(Give your answer in decimal).

$a =$

(ii) 图中的圆之圆心为 O， 半径为 1，A 和 B 是圆形上的点。
已知 $\dfrac{\text{陰影部分}}{\text{沒有陰影部分}}=\dfrac{\pi-2}{3\pi+2}$ 且 $\angle AOB=b^\circ$，求 b 的数值。

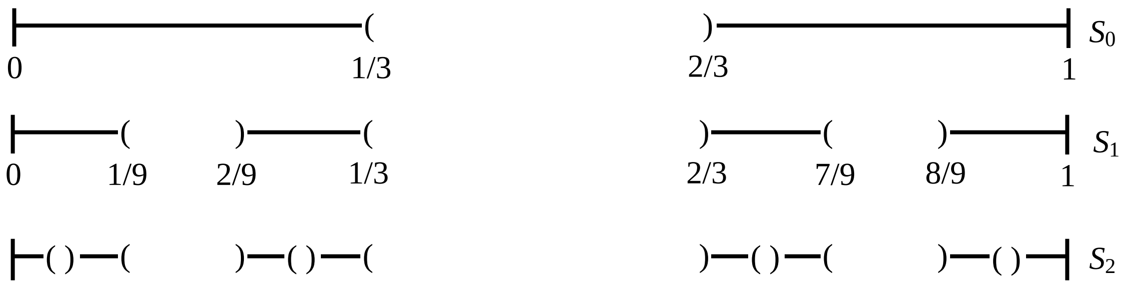
The circle in the figure has centre O and radius 1, A and B are points on the circle. Given that $\dfrac{\text{Area of shaded part}}{\text{Area of unshaded part}}=\dfrac{\pi-2}{3\pi+2}$ and $\angle AOB=b^\circ$, find the value of b.



$b =$

(iii) 图形 S_0 , S_1 , S_2 , …… 用以下方法构成。把线段 [0, 1] 的中间三分之一取去，得到 S_0，把 S_0 的两条组成线段，每段的中间三分之一取去，得到 S_1，把 S_1 的四条组成线段，每段的中间三分之一取去，得到 S_2。 S_3、S_4 …… 等用类似方法获得。求在构成 S_5 的过程中取去的线段的总长度 c。 (答案以分数表示)。

A sequence of figures S_0 , S_1 , S_2 , …… are constructed as follows. S_0 is obtained by removing the middle third of the interval [0, 1] ; S_1 by removing the middle third of each of the two intervals in S_0 ; S_2 by removing the middle third of each of the two intervals in S_1 . S_3 , S_4 , …… are obtained similarly. Find the total length c of the intervals removed in the construction of S_5 . (Give your answer in fraction).



$c =$

(iv) 把所有整数用下表的方法编码。若编码 101 至 200 的所有整数之和为 d ，求 d 的数值。

All integers are coded as shown in the following table. If the sum of all integers coded from 101 to 200 is d , find the value of d .

整数 Integer	…	…	–3	–2	–1	0	1	2	3	…	…
编码 Code	…	…	7	5	3	1	2	4	6	…	…

$d =$

Hong Kong Mathematics Olympiad (1997 – 98)

Final Event 5 (Group)

香港数学竞赛 **(1997 – 98)**

决赛项目 **5 (**团体**)**

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
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(i) 若 $1\times2\times3+2\times3\times4+3\times4\times5+\cdots+10\times11\times12=a$，求 a 的数值。

If $1\times2\times3+2\times3\times4+3\times4\times5+\cdots+10\times11\times12=a$, find the value of a.

$a =$

(ii) 已知 $5^x+5^{-x}=3$。若 $5^{3x}+5^{-3x}=b$，求 b 的数值。

Given that $5^x+5^{-x}=3$. If $5^{3x}+5^{-3x}=b$, find the value of b.

$b =$

(iii) 已知二次方程 $x^2+mx+n=0$ 的两根为 98 和 99，且 $y=x^2+mx+n$。若 x 取 0 、 1 、 2 、 … 、 100，则有 c 个 y 的数值能被 6 整除。求 c 的数值。

Given that the roots of quadratic equation $x^2+mx+n=0$ are 98 and 99 and $y=x^2+mx+n$. If x takes on the values of 0 , 1 , 2 , … , 100 , then there are c values of y that can be divisible by 6 . Find the value of c.

$c =$

(iv) 在图中，$ABCD$ 为一正方形，$BF \parallel AC$，且 $AEFC$ 为一菱形。若 $\angle EAC = d^\circ$，求 d 的数值。

In the figure, $ABCD$ is a square , $BF \parallel AC$, and $AEFC$ is a rhombus. If $\angle EAC = d^\circ$, find the value of d.

$d =$